

# The Implementation of Smoothly Moving Boundaries in 2D and 3D TLM Simulations

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## Abstract

In this paper a novel technique for arbitrary boundary positioning in TLM networks will be described. This capability removes the restriction that dimensions of TLM models can only be integer multiples of the mesh parameter and allows superior boundary resolution. Since the position of boundaries can be continuously varied even during a simulation, this feature can model moving boundaries for time domain optimization and phenomena such as the Doppler effect.

## 1 Introduction

The accurate modeling of waveguide components, discontinuities and junctions requires a precision in the positioning of boundaries that is identical to, or better than the manufacturing tolerances. In traditional TLM models of electromagnetic structures, boundaries can only be placed either across the nodes or halfway between nodes. Unless all dimensions of the structure are integer multiples of  $\Delta l/2$  the mesh parameter would have to be very small indeed, leading to unacceptable computational requirements. Similar considerations apply when curved boundaries with very small radii of curvature must be modeled. It is therefore important to provide for arbitrary positioning of walls. A method for changing the position of boundaries in 2D TLM through modification of the impulse scattering matrix of boundary nodes has been described already in 1973 by Johns [1] who, at the time, thought that the advantage of this procedure over stepped contour (Manhattan-style) modeling was too small to warrant the additional complexity of the algorithm. However, this is not true when analyzing narrowband waveguide components such as filters.

## 2 Boundary Extension by Reactive Elements

In Johns' concept of arbitrary wall positioning in 2D TLM [1] a boundary branch which has a length different from  $\Delta l/2$  is simply replaced by an equivalent branch of length  $\Delta l/2$  having an identical input admittance. This ensures synchronism but requires a different characteristic admittance for the boundary branch and hence, a modification of the impulse scattering matrix of the boundary node.

The method proposed in this paper leaves the impulse scattering matrix of the boundary nodes intact, but replaces the single boundary reflection coefficient by a recursive reflection algorithm which functions as follows.

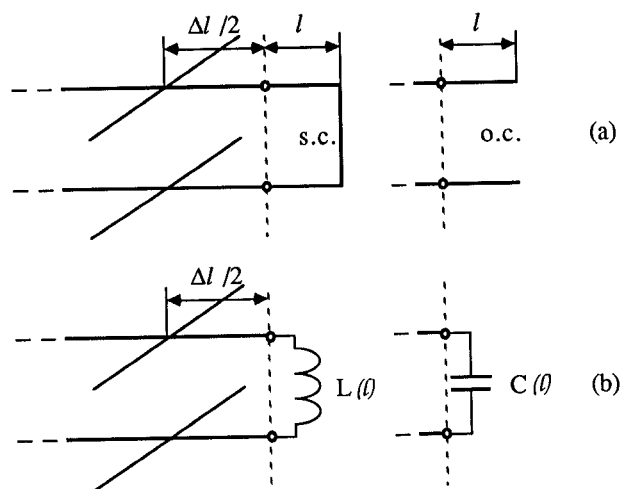


Fig. 1 Extension of short and open-circuit boundaries in a TLM mesh, and their representation by equivalent reactances.

Assume that we wish to position a reflecting boundary (electric or magnetic wall) at a distance  $\Delta l/2 + l$  from a node as shown in Fig. 1, where  $l$  is an arbitrary shift in the boundary position beyond the standard distance  $\Delta l/2$  for which the node scattering matrix has been defined. In fact, this amounts to terminating the regular  $\Delta l/2$  long boundary branch in a short- or open-circuited transmission line section with the normalized input reactance  $z_i$ ,

$$z_i = jx_i = \frac{j\omega L}{Z_0} = j\tan\beta l \quad \text{for an electric wall, and} \quad (1)$$

$$z_i = jx_i = \frac{1}{j\omega C Z_0} = \frac{1}{j\tan\beta l} \quad \text{for a magnetic wall.}$$

As long as the excess length  $l$  is much smaller than the wavelength (or  $\beta l \ll 1$ ), the inductance or capacitance of the

branch extension can be considered independent of frequency, since  $\tan\beta l \approx \beta l$ , yielding

$$L \approx \frac{Z_0 l}{c} \quad \text{for an electric wall, and} \quad (2)$$

$$C \approx \frac{l}{cZ_0} \quad \text{for a magnetic wall.}$$

where the propagation velocity on the TLM mesh lines is taken as  $c$ .

It is now possible to write the differential equation relating voltage and current at the input of the reactive stubs in terms of the incident and reflected impulses, and to replace this differential equation by a difference equation. This results in the following general recursive formula:

$${}_k V^i = \rho \frac{\kappa - 1}{\kappa + 1} {}_k V^r + \frac{\kappa}{\kappa + 1} (\rho {}_{k-1} V^r + {}_{k-1} V^i) \quad (3)$$

where  $\rho = +1$  for a magnetic wall and  $\rho = -1$  for an electric wall.  $\kappa$  is equal to  $\frac{2l}{\Delta l}$  in the 3D TLM case, and  $\frac{\sqrt{2}l}{\Delta l}$  in the 2D TLM case. Eq. (3) indicates that the present impulse reflected from the boundary in the reference plane at  $\Delta l/2$  depends on the present incident impulse as well as on the previous incident and reflected impulses, which need to be stored. This recursive algorithm amounts to a numerical procedure for integrating the differential equation describing the behavior of the reactive stub in the time domain.

### 3 Verification of Results

The accuracy of the above algorithm has been validated by performing extensive simulations of structures most sensitive to small variations in their dimensions, namely quarterwave and halfwave resonators as shown in Fig. 2. One of the walls was made movable by application of Eq. (3), and 3D-TLM results obtained with the condensed node scheme [2] for the resonant frequencies were compared with accurate analytical values. Fig. 3 demonstrate the results. Data obtained for higher order modes yield information on the accuracy of the algorithm as a function of the incident angle. It appears that the error margin is largest for incident angles around 45 degrees.

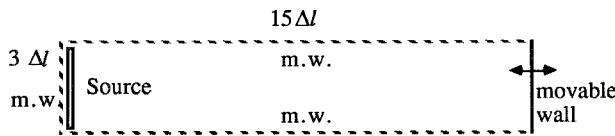


Fig. 2 Quarter-wave resonator with movable sidewall for validation of the proposed algorithm.

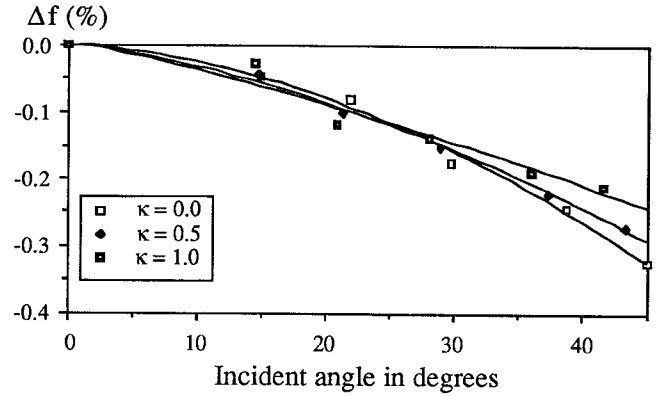


Fig. 3 Relative error in resonant frequency of the resonator shown in Fig. 2 as a function of the angle of incidence for the three values of  $\kappa = 2l/\Delta l$ .

### 4 Conclusion

The new technique presented in this paper effectively removes the restriction that dimensions of TLM models can only be integer multiples of the mesh parameter. It thus considerably improves the flexibility of TLM modeling of microwave/millimeter-wave/optical components by freeing the modeler from the "Manhattan-style" approximation of curved boundaries and by improving the geometrical resolution without increase in computational expenditure. Since the position parameter  $\kappa = 2l/\Delta l$  can be varied in arbitrarily small increments between computational steps, this feature can be used to model moving boundaries and allows optimization in the time domain through modification of structure geometry during a simulation. Also, the direct visualization of phenomena such as the Doppler effect becomes feasible.

### REFERENCES

- [1] P. Johns, "Transient Analysis of Waveguides with Curved Boundaries", Electronics Letters, vol. 9, no. 21, 18th Oct. 1973.
- [2] P.B. Johns, "A Symmetrical Condensed Node for the TLM Method", IEEE Trans. Microwave Theory Tech., vol. MTT-35, no. 4, pp. 370-377, April 1987.